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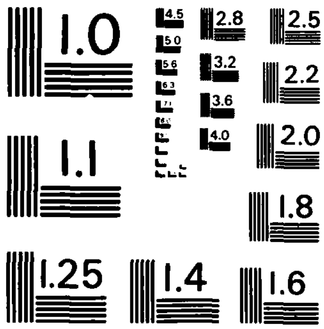

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DETERMINATION OF NECESSARY COMMANDS FOR THE MOTION OF A ROCKET ON A GIVEN LINEAR TRAJECTORY IN THE ATMOSPHERE

by

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DETERMINATION OF NECESSARY COMMANDS FOR THE MOTION OF A ROCKET ON A GIVEN LINEAR TRAJECTORY IN THE ATMOSPHERE

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The variation in time of the attack angle of the control surfaces of a rocket so that it is moving on a given linear trajectory in the atmosphere is determined in this paper. Variation of aerodynamic coefficients with the Mach number is considered as well as the variation in time (in the fuel combustion stage) of the moments of inertia and the position of the center of mass of the rocket. Air density and sound velocity are assumed variable with the altitude. A numerical calculation example solved with the algorithm presented in this paper is shown in the end.

1. MAIN NOTATIONS USED

Oxy is the reference system stationary with respect to the Earth, with the Ox axis along with the linear trajectory required (fig. 1)

θ	- the angle of the linear trajectory in the horizontal direction
t	- the time measured from the moment the fuel combustion begins
$V = \frac{dx}{dt} = \dot{x}$	- the velocity of the center of mass of the rocket
g	- acceleration of gravity
h	- the flight altitude of the rocket
ρ	- the density of the atmosphere at flight altitude
a	- sound velocity in atmosphere at flight altitude
M	- Mach number for the flight

$O'\xi\eta$ - reference system stationary with respect to the body of the rocket and whose axis $0'\xi$ coincides with the axis of symmetry of the rocket (Fig. 1),

m - mass of the rocket, variable during the fuel combustion stage

J - moment of inertia of the rocket with respect to the rolling axis (which passes through the center of mass), variable while the engine is running

$Q = -\frac{\dot{m}}{m_0}$ - flow of gases in the engine, adimensional

ξ_C - abscissa of the center of mass of the rocket in the system $0'\xi\eta$;

ξ_F - abscissa, in the system $0'\xi\eta$, of the point F with respect to which the momentary aerodynamic coefficients are calculated

$\xi = \xi_C - \xi_F$;

T - traction force of the engine of the rocket

t_a - the time the fuel is burnt (including t_{tr})

t_{tr} - time of transition from active stage with engine running to passive stage

S - reference surface to which the aerodynamic coefficients are referred

l - the reference length to which the coefficient of the moment of rolling is referred

α - the angle of incidence of the rocket (the angle between the direction of the velocity and the axis of symmetry of the body of the rocket) (Fig. 1);

β - the angle of attack of the surfaces of command (fig. 1)

$q = \dot{\alpha}$ - angular rolling velocity of the rocket

$\varepsilon = \ddot{\alpha}$ - angular rolling acceleration of the rocket

$\hat{q} = \frac{ql}{2V}$ - rolling velocity of the rocket, adimensional

L - aerodynamic lift force of the rocket

D - aerodynamic drag force of the rocket

M_m - aerodynamic rolling moment of the rocket with respect to point F

$C_L = \frac{L}{\frac{1}{2}\rho V^2 S}$ - aerodynamic lift coefficient

$C_D = \frac{D}{\frac{1}{2}\rho V^2 S}$ - aerodynamic drag coefficient

C_{D_0} - aerodynamic drag coefficient at zero incidence

$C_{D_{fd}}$ - base pressure drag coefficient of the rocket

k_D - constant in the formula of C_D

$C_m = \frac{M_m}{\frac{1}{2}\rho V^2 S l}$ - rolling moment coefficient with respect to point F

$C_L^{\alpha} = \frac{dC_L}{d\alpha}, C_m^{\alpha} = \frac{dC_m}{d\alpha}$ - derivatives of aerodynamic lift and rolling moment coefficients with respect to the incidence of the rocket

$C_L^{\beta} = \frac{dC_L}{d\beta}, C_m^{\beta} = \frac{dC_m}{d\beta}$ - derivatives of aerodynamic lift and rolling moment coefficients with respect to the angle of attack of the command surfaces

$C_m^{\hat{q}} = \frac{dC_m}{d\hat{q}}$ - derivative of the rolling moment coefficient with respect to the adimensional rolling velocity (coefficient of rolling reduction)

DERIVATION SIGNS AND INDEXES

$(\dot{\ })$ derivative with respect to time

$(\ )_o$ value at initial moment $t_o = o$

$(\ )_f$ value at final moment t_f

2. INTRODUCTION

Solving a problem of the dynamics of the flight of a rocket in the atmosphere is the scope of this paper, a problem often met in certain practical cases like the determination of the variation law of the attack angle of the command surfaces so that the rocket will follow a given linear trajectory in the atmosphere which it must pursue both in the active stage of the flight (with the engine running) and passive stage (without traction force). The variation of the aerodynamic coefficients with the Mach number is being evaluated as well as the variation in time, with respect to fuel consumption, of the inertial moments and of the position of the center of mass of the rocket. These are considered as numerical functions with respect to the Mach number and time, respectively, being determined either theoretically or experimentally.

The traction force of the rocket is considered constant in the fuel combustion stage (active stage) and it is assumed that it is linearly decreasing in time to zero in the short transition period from active to passive stage.

3. THE DIFFERENTIAL EQUATIONS FOR THE MOTION OF THE ROCKET

It is assumed that the flight distances are short compared to the radius of the Earth so that the acceleration of gravity is considered to be constant in magnitude and direction ($\bar{g}$ = const.) and that the rocket motion takes place on a linear trajectory tilted at the angle θ from the horizontal direction, situated in the vertical plane defined by the direction of the initial velocity of the center of mass of the rocket and the vector of the acceleration of gravity (fig. 1).

The differential equations of the motion of the rocket in this plane are:

$$T\cos\alpha - D - mg\sin\theta = m\ddot{x}, \tag{1.a}$$

$$T\sin\alpha + L - mg\cos\theta = 0, \tag{1.b}$$

$$M_m - \xi(L\cos\alpha + D\sin\alpha) = J\ddot{\alpha}. \tag{1.c}$$

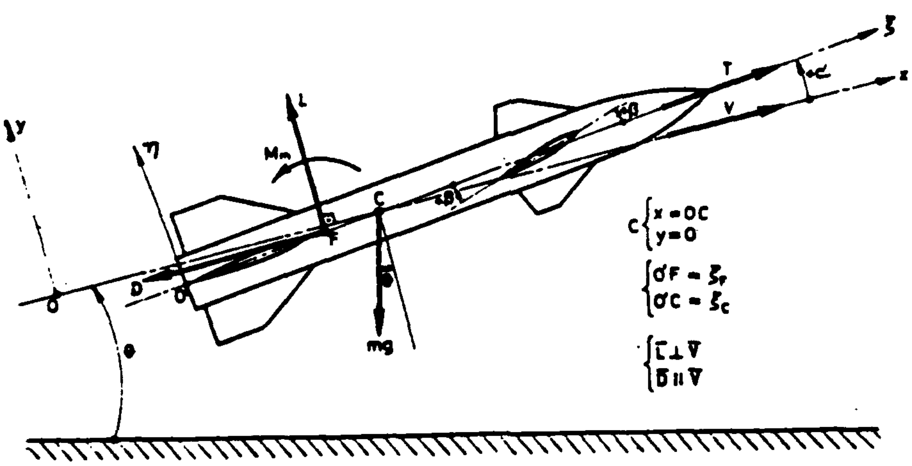

Fig. 1

In these equations the mass of the rocket m has a linear variation with respect to time in the active stage, being constant in the passive stage

$$m = \begin{cases} m_0(1 - Qt) & \text{for} \quad t \in [0, t_a), \quad (2.a) \\ m_f = \text{const.} & \text{for} \quad t > t_a, \quad (2.b) \end{cases}$$

where the adimensional flow of burnt gases is constant and given by:

$$Q = -\frac{\dot{m}}{m_0} = \frac{1}{t_a}\left(1 - \frac{m_f}{m_0}\right) = \text{const.} \quad (3)$$

The traction force T is considered constant in the active stage, and in the transition period it decreases continuously in time to cancellation:

$$T = \begin{cases} T_0 = \text{const.} & \text{for} \quad t \in [0, t_a - t_{tr}], \\ T_{tr}(t) & \text{for} \quad t \in [t_a - t_{tr}, t_a], \\ 0 & \text{for} \quad t > t_a. \end{cases} \quad (4)$$

A linear variation of the traction force in time for the transition period was considered in the numerical example at the end of this paper:

$$T_{tr}(t) = T_0\left[1 - \frac{t - (t_a - t_{tr})}{t_{tr}}\right]. \quad (5)$$

The aerodynamic forces and moments are given by:

$$L = \frac{1}{2}\rho V^2 S C_L; \; C_L = \alpha C_L^\alpha + \beta C_L^\beta, \quad (6.a)$$

$$D = \frac{1}{2}\rho V^2 S C_D, \quad (6.b)$$

$$M_m = \frac{1}{2}\rho V^2 S l C_m; \; C_m = \alpha C_m^\alpha + \beta C_m^\beta + \hat{q} C_m^{\hat{q}}, \quad (6.c)$$

where the drag coefficient is given by:

$$C_D = \begin{cases} C_{D_0} + k_D\alpha^2 & \text{for} \quad \in [0, t_a], \quad (7.a) \\ C_{D_0} + k_D\alpha^2 + C_{D/a} & \text{for} \quad > t_a, \quad (7.b) \end{cases}$$

where C_{dfd} is the base pressure drag coefficient which appears when the engine ceases to function.

In (6) the other derivatives of the aerodynamic coefficients (like $C_L^{\hat{q}}$) were assumed negligible. Also, considering a canard configuration for the rocket, with the command surfaces placed in the front, the effect of the vortex created by them on the wings could be neglected.

The aerodynamic coefficients and their derivatives are known functions with respect to Mach number. Thus, we assume their values known for $C_{D_0}(M_i), k_B(M_i), C_{Dfd}(M_i), C_L^{\alpha}(M_i), C_m^{\alpha}(M_i), C_L^{\delta}(M_i), C_m^{\delta}(M_i), C_m^{\hat{q}}(M_i)$ at different Mach numbers M_i, $i \in \{1, 2, 3, \ldots, n_M\}$, situated in the range of velocities of flight.

Also assumed known are the values at different moments t_i of the distance ξ between the center of the mass of the rocket C and the fixed point F with respect to which the aerodynamic rolling moment M_m is determined, as well as the values of the moment of inertia J of the rocket with respect to the rolling axis that passes through the center of mass, that is $\xi(t_i)$ and $J(t_i)$ are known at different moments t_i, $i \in \{1,2,3,\ldots, n_t\}$, during the fuel combustion stage $[o, t_\alpha]$.

In order to determine the values of the aerodynamic coefficients and of their derivatives for a Mach number M as well as the values of the moment of inertia J and the distance ξ which defines the position of the center of mass at a time t, a numerical method of the Sline type has been used.

Atmosphere density ρ and sound velocity in the atmosphere are variable with respect to the altitude of flight h of the rocket and are given by

$$h = h_0 + x \sin \theta, \qquad (8)$$

in which h_o is the altitude at take-off.

Substituting in the system of differential equations (1) the expression of the mass m given by (2.a) and the expressions of the aerodynamic coefficients given by (6) and (7.a) and those of the aerodynamic forces and moments, this system becomes:

$$T\cos\alpha - \frac{1}{2}\rho V^2 S(C_{D_0} + k_D\alpha^2) - m_0(1-Qt)\,g\sin\theta = m_0(1-Qt)\,\ddot{x}, \qquad (9.a)$$

$$T\sin\alpha + \frac{1}{2}\rho V^2 S(\alpha C_L^\alpha + \beta C_L^\beta) - m_0(1-Qt)\,g\cos\theta = 0, \qquad (9.b)$$

$$\frac{1}{2}\rho V^2 Sl(\alpha C_m^\alpha + \beta C_m^\beta + \hat{q}C_m^{\hat{q}}) - \frac{1}{2}\rho V^2 S\xi[(\alpha C_L^\alpha + \beta C_L^\beta)\cos\alpha +$$
$$+ (C_{D_0} + k_D\alpha^2)\sin\alpha] = J\ddot{\alpha}. \qquad (9.c)$$

4. INTEGRATION OF THE DIFFERENTIAL EQUATIONS OF MOTION OF THE ROCKET

The system of differential equations (9) is a nonlinear system whose integration is possible only by numerical methods. For this purpose we obtain the expression of β from (9.c):

$$\beta = \frac{1}{C_m^\beta - \frac{\xi}{l}C_L^\beta\cos\alpha}\left[\frac{J}{\frac{1}{2}\rho V^2 Sl}\ddot{\alpha} - \alpha\left(C_m^\alpha - \frac{\xi}{l}C_L^\alpha\cos\alpha\right) - \hat{q}C_m^{\hat{q}} + \frac{\xi}{l}(C_{D_0} + k_D\alpha^2)\sin\alpha\right]. \qquad (10)$$

Substituting this expression in (9.b) and using the notations

$$T^* = \frac{T}{\frac{1}{2}\rho V^2 S}; \quad G^* = \frac{m\,g\cos\theta}{\frac{1}{2}\rho V^2 S}; \quad J^* = \frac{J}{\frac{1}{2}\rho V^2 Sl}. \qquad (11)$$

the following equation is obtained:

$$(T^*\sin\alpha + \alpha C_L^\alpha - G^*)\left(C_m^\beta - \frac{\xi}{l}C_L^\beta\cos\alpha\right) + C_L^\beta\left[J^*\ddot{\alpha} - \alpha\left(C_m^\alpha - \frac{\xi}{l}C_L^\alpha\cos\alpha\right) - \dot{\alpha}\frac{l}{2V}C_m^{\hat{q}} + \frac{\xi}{l}(C_{D_0} + k_D\alpha^2)\sin\alpha\right] = 0, \qquad (12)$$

which, together with (9.a) is a nonlinear system of differential equations of second order.

Then, introducing the supplementary variables $V = \dot{x}$ and $q = \dot{\alpha}$, the system of differential equations (9a and 12) of second order becomes a system of first order

$$\dot{x} = V, \tag{13.a}$$

$$\dot{V} = g\left[\frac{T^* \cos\alpha - (C_{D_0} + k_D\alpha^2)}{G^*}\cos\theta - \sin\theta\right], \tag{13.b}$$

$$\dot{\alpha} = q,$$

$$\dot{q} = \frac{1}{J^*}\left[\frac{G^* - T^*\sin\alpha - \alpha C_L^\alpha}{C_L^\beta}\left(C_m^\beta - \frac{\xi}{l}C_L^\beta\cos\alpha\right) + \right. \tag{13.c}$$

$$\left. + \left(C_m^\alpha - \frac{\xi}{l}C_L^\alpha\cos\alpha\right)\alpha + \frac{l}{2V}C_m^{\hat{q}}q - \frac{\xi}{l}(C_{D_0} + k_D\alpha^2)\sin\alpha\right] \tag{13.d}$$

This nonlinear system of differential equations of first order can be, in principle, numerically integrated using an adequate method such as the Runge-Kutta method. At every step the value of the attack angle β can be determined, by using (10).

It is noted though, that in many cases, the derivative C_L^β of the drag coefficient with respect to the attack angle β has very small values, and sometimes people tend to neglect it, which leads to great errors when using the integration method above mentioned (C_L^β is in the equation (13.d) in the denominator) and even making it impossible to use the respective method (in the case that C_L^β is negligible).

For this reason, a procedure which can be applied even for very small values of the derivatives C_L^β and even for $C_L^\beta \approx 0$ will be shown here.

This procedure consists of using at the same time the Runge-Kutta method for the numerical integration of the system of differential equations (13.a) and (13.b) and a method with finite

differences in order to integer the differential equation (12) which is equivalent with (13.c) and (13.d). Regressive differences of second order are used for this purpose in order to obtain the derivatives $\dot{\alpha}$ and $\ddot{\alpha}$ which appear in equation (12):

$$\dot{\alpha}_i = b_1 \alpha_i + b_2, \tag{14.a}$$

$$\ddot{\alpha}_i = c_1 \alpha_i + c_2, \tag{14.b}$$

in which:

$$b_1 = \frac{3}{2 \cdot \Delta t}; \; b_2 = \frac{1}{2 \cdot \Delta t} (- 4\alpha_{i-1} + \alpha_{i-2}), \tag{15.a}$$

$$c_1 = \frac{2}{(\Delta t)^2}; \; c_2 = \frac{1}{(\Delta t)^2} (- 5\alpha_{i-1} + 4\alpha_{i-2} - \alpha_{i-3}), \tag{15.b}$$

and Δt represents the integration step with respect to time.

For the integration step i = 1, at which only the values of α marked i - 1 and for the step i = 2 for which only the values marked i -1 and i - 2 are known, the values of b_1, b_2, c_1, c_2, from (14) can be calculated with the following formula *

$$b_1 = \frac{3}{2 \cdot \Delta t}; \; b_2 = \frac{1}{2 \cdot \Delta t} \left[- 3\alpha_{i-1} - \Delta t \cdot \dot{\alpha}_{i-1} + \frac{(\Delta t)^2}{2} \ddot{\alpha}_{i-1} \right], \tag{16.a}$$

$$c_1 = \frac{2}{(\Delta t)^2}; \; c_2 = - \frac{2}{(\Delta t)^2} (\alpha_{i-1} + \Delta t \cdot \dot{\alpha}_{i-1}). \tag{16.b}$$

It is noted here that the initial conditions α_0, $\dot{\alpha}_0$ $\ddot{\alpha}_0$ are known.

From (12) the next relation can be obtained with which the value α_i of the corresponding integration step i can be calculated:

$$(T^* \sin \alpha_i + C_L^\alpha \alpha_i - G^*) \left(C_m^\delta - \frac{\xi}{l} C_L^\delta \cos \alpha_i \right) +$$

$$+ C_L^\delta \left[J^*(c_1 \alpha_i + c_2) - \left(C_m^\alpha - \frac{\xi}{l} C_L^\alpha \cos \alpha_i \right) \alpha_i - \right.$$

$$\left. - \frac{l}{2V} C_m^{\dot{\alpha}} (b_1 \alpha_i + b_2) + \frac{\xi}{l} (C_{D_0} + k_D \alpha_i^2) \sin \alpha_i \right] = 0. \tag{17}$$

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* The equations (16) are obtained by setting α_{i-2}, α_{i-3} as functions of α_{i-1}, $\dot{\alpha}_{i-1}$, $\ddot{\alpha}_{i-1}$ from the formula with regressive differences of second and first order which yield $\dot{\alpha}_{i-1}$, $\ddot{\alpha}_{i-1}$ respectively by substituting the result in the equations (15).

In order to determine the unknown value of α_i from the transcendental equation (17) a method of successive approximations is used. For this purpose the equation (17) is written in the form

$$\alpha_i = f(\alpha_i), \tag{18.a}$$

where

$$f(\alpha_i) = \frac{E - F\cos\alpha_i + (A - B\cos\alpha_i + C\alpha_i^2)(\alpha_i - \sin\alpha_i)}{D + (A - B\cos\alpha_i + C\alpha_i^2)}, \tag{18.b}$$

and

$$A = T^* C_m^3 + \frac{\xi}{l} C_L^3 C_{D_0}, \tag{19.a}$$

$$B = \frac{\xi}{l} T^* C_L^3, \tag{19.b}$$

$$C = \frac{\xi}{l} k_D C_L^3, \tag{19.c}$$

$$D = C_m^3 C_L^\alpha - C_L^3 \left(C_m^\alpha - J^* c_1 + \frac{l}{2V} C_m^{\hat{q}} b_1 \right), \tag{19.d}$$

$$E = G^* C_m^3 - C_L^3 \left(J^* c_2 - \frac{l}{2V} C_m^{\hat{q}} b_2 \right), \tag{19.e}$$

$$F = \frac{\xi}{l} G^* C_L^3. \tag{19.f}$$

It can be shown that for the usual range of incidences (- 12°,+ 12°), that is (- 0.2, + 0.2 rad) and for usual values of aerodynamic, gemoetrical and inertial values of the rocket, the function $f(\alpha_i)$ defined by (18.b) satisfies the conditions

$$\sup |f'_{\alpha_i}(\alpha_i)| = \lambda, \quad \lambda < 1, \qquad \alpha_i \in (-0{,}2, +0{,}2). \tag{20}$$

In these conditions, the application f is a contraction and thus, according to the Picard - Banach theorem, the fixed point α_i of the function $f(\alpha_i)$, that is the root of the equation (17) exists and is unique, and

it can be determined using an iteration method with the help of the series generated by the relation:

$$\alpha_{i,n} = \frac{E - F\cos\alpha_{i,n-1} + (A - B\cos\alpha_{i,n-1} - C\alpha_{i,n-1}^2)(\alpha_{i,n-1} - \sin\alpha_{i,n-1})}{D + (A - B\cos\alpha_{i,n-1} + C\alpha_{i,n-1}^2)}. \quad (21)$$

It can be demonstrated that, no matter what α_{i0} is within the interval (- 0.2 + 0.2) rad, the series (21) is convergent towards the fixed point α_i and the error, which may occur $d(\alpha_i, \alpha_{i,n})$ if instead of the root α_i the value $\alpha_{i,n}$ is taken, is given by the relation

$$d(\alpha_i, \alpha_{i,n}) \leq \frac{\lambda^n}{1-\lambda} \cdot d(\alpha_{i,1}, \alpha_{i,0}). \quad (22)$$

As a starting number $\alpha_{i,0}$ in the iteration to determine the angle α_i, the value of the incidence at the previous step can be used, that is:

$$\alpha_{i,0} = \alpha_{i-1} \approx \alpha_{i-1,n}. \quad (23)$$

It is mentioned that by writing equation (17) under the form (18), the iteration procedure used to determine the angle α_i becomes very rapidly convergent in all the analyzed cases.

With respect to the value α_i so determined the attack angle of the command surfaces can be determined at every β integration step. (The values for $\dot{\alpha}_i = q_i$ and $\dot{\alpha}_i$ which occur in (10) are given by (14)).

a) ESTABLISHING THE INITIAL CONDITIONS OF THE MOTION

In order to start the numerical integration procedure, the initial conditions of the motion have to be known, that is, the following kinematic parameters at launching time: h_o, x_0, V_0, α_0., q_0.

Assuming that the launching takes place from a fixed ramp or from a mobile ramp (i.e, a plane) whose motion is known, then the initial parameters h_0, x_0, V_0, $q_0 = \dot{\alpha}_0$, are known as well as the angular acceleration $\varepsilon_0 = \ddot{\alpha}_0$.

With regards to the initial value α_0 and β_0, these must be determined in such a way so that the rocket follows the linear trajectory from launching. It is mentioned that in the present paper it is assumed that the rocket moves on a linear trajectory straight from launching. In a future paper the case when the rocket first moves on a curved trajectory and then starts a linear motion will be considered.

To determine α_0 the initial conditions of the motion are substituted in (12). A transcendental equation in α_0 is obtained which is solved using the same method of successive approximations as in the case of equation (17). For this purpose the equation is put under the form

$$\alpha_0 = f_0(\alpha_0), \tag{24.a}$$

in which

$$f_0(\alpha_0) = \frac{E_0 - F_0 \cos \alpha_0 + (A_0 - B_0 \cos \alpha_0 + C_0 \alpha_0^2)(\alpha_0 - \sin \alpha_0)}{D_0 + (A_0 - B_0 \cos \alpha_0 + C_0 \alpha_0^2)} \tag{24.b}$$

and where:

$$A_0 = T_0^* C_{m_0}^{\beta} + \frac{\xi_0}{l} C_{D_{00}} C_{L_0}^{\beta}, \tag{25.a}$$

$$B_0 = T_0^* \frac{\xi_0}{l} C_{L_0}^{\beta}, \tag{25.b}$$

$$C_0 = \frac{\xi_0}{l} k_{D_0} C_{L_0}^{\beta}, \tag{25.c}$$

$$D_0 = C_{L_0}^{\alpha} C_{m_0}^{\beta} - C_{L_0}^{\beta} C_{m_0}^{\alpha}, \tag{25.d}$$

$$E_0 = G_0^* C_{m_0}^3 - C_{L_0}^3 \left(J_0^* \varepsilon_0 - \frac{l}{2 \Gamma_0} q_0 C_{m_0}^{\hat{1}} \right), \qquad (25.e)$$

$$F_0 = G_0^* \frac{\xi_0}{l} C_{L_0}^3. \qquad (25.f)$$

It can be demonstrated that in this case also $f_0(\alpha_0)$ is a contraction so that the Picard - Banach theorem can be used for the convergent series of successive approximations:

$$\alpha_{0.n} = \frac{E_0 - F_0 \cos \alpha_{0.n-1} + (A_0 - B_0 \cos \alpha_{0.n-1} + C_0 \alpha_{0.n-1}^2)(\alpha_{0.n-1} - \sin \alpha_{0.n-1})}{D_0 + (A_0 - B_0 \cos \alpha_{0.n-1} + C_0 \alpha_{0.n-1}^2)}. \qquad (26)$$

The starting value for the iteration to determine α_0 is obtained for $n = 1$ and $\alpha_{0.0} \cong 0$ in (26):

$$\alpha_{0.1} = \frac{E_0 - F_0}{D_0 + A_0 - B_0}. \qquad (27)$$

The iteration process, which is rapidly convergent, stops at the number n which yields α_0 with the desired accuracy ($\alpha_0 \approx \alpha_{0.n}$).

Having α_0, the initial attack angle β_0 can be calculated with (10), in which the initial values α_0, $\dot{\alpha}_0 = q_0$, $\ddot{\alpha}_0 = \varepsilon_0$ are substituted, as well as the initial values of the aerodynamic coefficients of other parameters variable in time, such as m_0, ξ_0, J_0, ρ_0 etc.

b) NUMERICAL INTEGRATION OF THE SYSTEM OF DIFFERENTIAL EQUATIONS OF MOTION

Knowing the initial conditions for the motion: h_0, x_0, V_0, α_0, q_0, the system of differential equations (13.a) and (13.b) can be integrated by one of the known numerical methods, i.e the Runge - Lutta - Gill method of the fourth order, which has the desired accuracy using a halving method for the integration step for which the calculation error is reduced to the imposed value.

The angle α of (13.b) is determined at every step using the iteration procedure above mentioned.

During the numerical integration, the aerodynamic coefficients and their derivatives are considered variable with Mach number being given by numerical values. Their values corresponding to the Mach number of the flight at a given moment are calculated using a Spline type of numerical interpolation method. A similar method can be used to determine at every integration step the moment of inertia J of the rocket and the coordinates ξ of the center of mass which also are variable with respect to time, being previously calculated and given by numerical values. They can be calculated also during the numerical integration at every step, if the relation between the fuel consumption and its distribution in the rocket body is known.

To calculate the thermodynamic parameters of the atmosphere (density and sound velocity) which are variable with the altitude h, a method of numerical interpolation can be used starting from their values in standard conditions or using analytical calculus relations as is the case in the numerical example presented at the end of this paper.

At every step of numerical integration which corresponds to a certain value of the time t, by the algorithm presented in part 4 the values of x, V, α, q, ε are determined, with which from (10) the value β of the attack angle is determined.

The succession in time of the values β of the attack angle represents the required command law for the rocket to move on a linear trajectory.

5. THE MOTION OF THE ROCKET IN THE PASSIVE STAGE

The rocket has to continue the linear motion $(t > t_a)$, after engine shut-off. The differential equations of the motion for this stage are obtained from the corresponding equations in the active stage in which:

$$T = 0, \quad J = J_f = \text{const.},$$

$$m = m_0 (1 - Qt_a) = m_f = \text{const.}, \; \xi = \xi_f = \text{const.}$$

and the bottom drag coefficient C_{Dfd} is added to the drag coefficient at zero lift C_{Do}.

For the transition period it can be assumed that for the time t_{tr} = (0.2 - 0.4) seconds the mass of the rocket decreases linearly in time as in the active stage, and the traction force for the transition period T_{tr} also decreases linearly from a value T_0 to zero according to (5). In order to find the differential equations of the motion for this stage, we substitute T with T_{tr} in the equations of the motion for the active stage. The same numerical methods will be used as for the active stage for integration of the differential equations of the motion of the rocket corresponding to the last two stages (transition and passive).

6. NUMERICAL EXAMPLE OF CALCULATION

In order to illustrate the above shown algorithm, the following numerical example is presented in which these data have been used:

- geometrical and inertial characteristics of the rocket:

$$S = 1\ \mathrm{m}^2, \qquad m_0 = 600\ \mathrm{kg},$$

$$l = 0{,}7\ \mathrm{m}, \qquad m_f = 400\ \mathrm{kg},$$

$$\xi_p = 1{,}8\ \mathrm{m}, \qquad J_0 = 124{,}53\ \mathrm{kg\ m}^2,$$

$$\xi_{c_0} = 1{,}86667\ \mathrm{m}, \qquad J_f = 100\ \mathrm{kg\ m}^2\ ;$$

$$\xi_{c_f} = 1{,}8\ \mathrm{m},$$

- the characteristics of the motor (three successive values have been considered for the traction force):

$$T_0 = 25\,000\ \mathrm{N}, \qquad t_a = 5\ \mathrm{s},$$

$$T_0 = 50\,000\ \mathrm{N}, \qquad t_{tr} = 0{,}4\ \mathrm{s};$$

$$T_0 = 100\,000\ \mathrm{N},$$

- the initial conditions of the motion:

$$h_0 = 1\,000 \text{ m}, \qquad V_0 = 200 \text{ m/s},$$
$$x_0 = 0, \qquad q_0 = \varepsilon_0 = 0;$$

- the slope of the linear trajectory

$$\theta = 15°;$$

- the required time for the rocket to maintain a given linear trajectory

$$t_f = 25 \text{ s}.$$

The position ξ_c of the center of the mass of the rocket and the moment of inertia J with respect to the rolling axis have been calculated as numerical functions of time considering a certain fuel distribution inside the rocket. Their variation in time is shown in fig. 2 and has been introduced in calculations as numerical form.

The variation of the aerodynamic coefficients and of their derivatives with respect to the Mach number is indicated, for the example given, in fig. 3, 4, 5. Their values have been introduced in calculations in numerical form. k_D C_l^α (1/rad) and $C_{Dfd} \approx 0$ were considered in this example. Air density, variable with the altitude h, was calculated by:

$$\rho = \rho_p \cdot e^{-k_\rho h}, \qquad (28)$$

where ρ_p is air density at sea level (ρ_p = 1.229 kg/m^{3}0), and $k\rho$ is a constant for which different values were taken in 1,000 m ranges, values calculated with the data derived from standard atmospheric values.

In order to calculate the sound velocity in the atmosphere the following expressions have been used:

$$a = a_p - k_s h \qquad \text{for} \quad h < 10\cdot 668 \text{ m},$$
$$a = 296{,}2656 \text{ m/s} \qquad \text{for} \quad h \geqslant 10\cdot 668 \text{ m}, \tag{29}$$

where a_p is sound velocity at sea level (a_p = 340.1568 m/s), k_s is a constant (k_s = 0.004114286 s^{-1}) and h is measured from sea level.

The results are shown in the diagrams of fig. 6 - 10 in which the distance x that the rocket cruised, the velocity of the rocket V, the Mach number of the flight M, the angle of incidence α and the attack angle β of the command surfaces were represented with respect to time for the three values considered for the traction force.

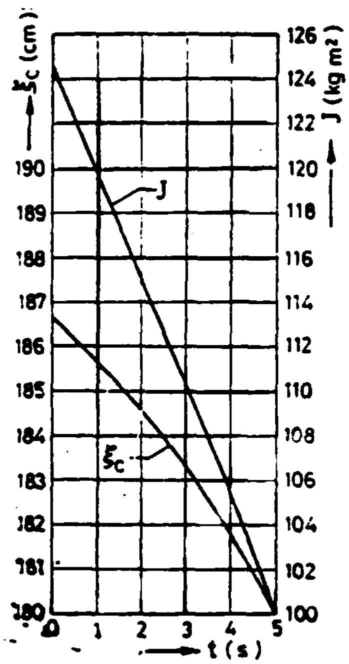


Fig. 2

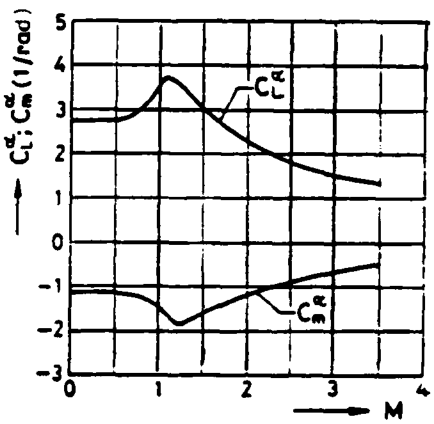
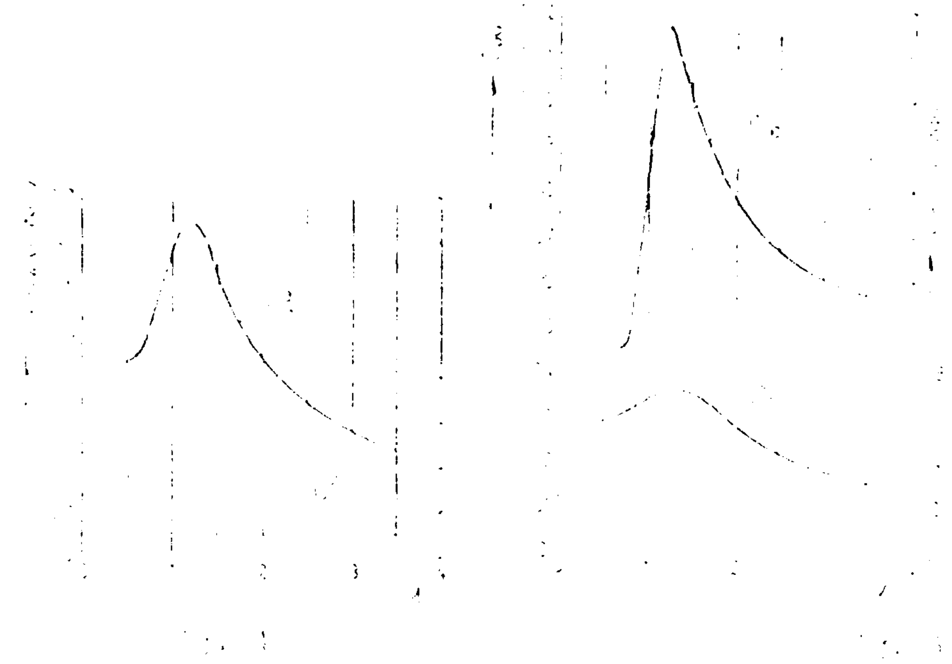
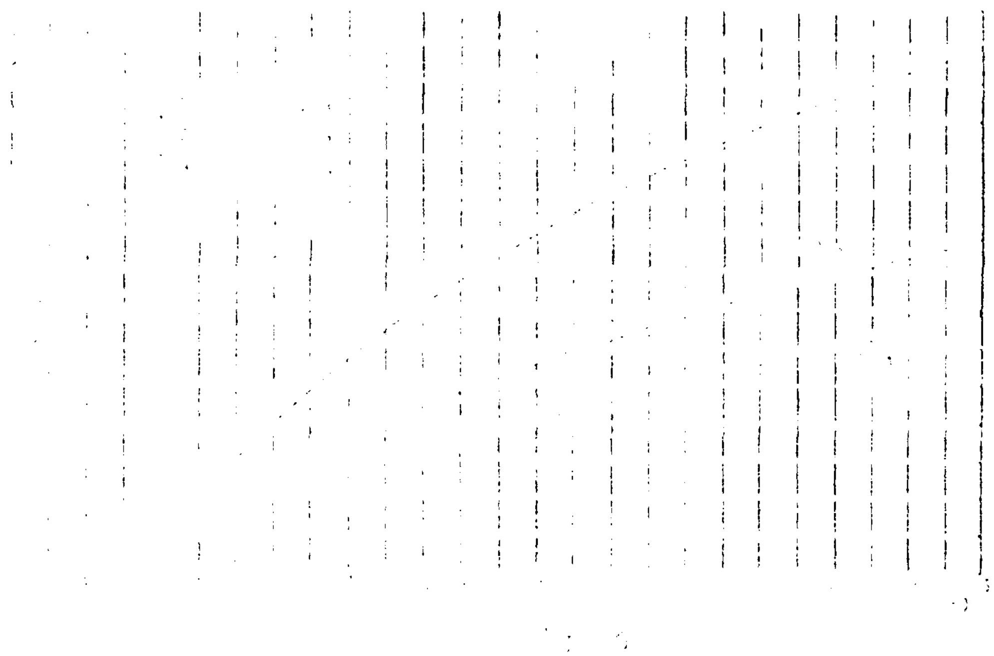


Fig. 3





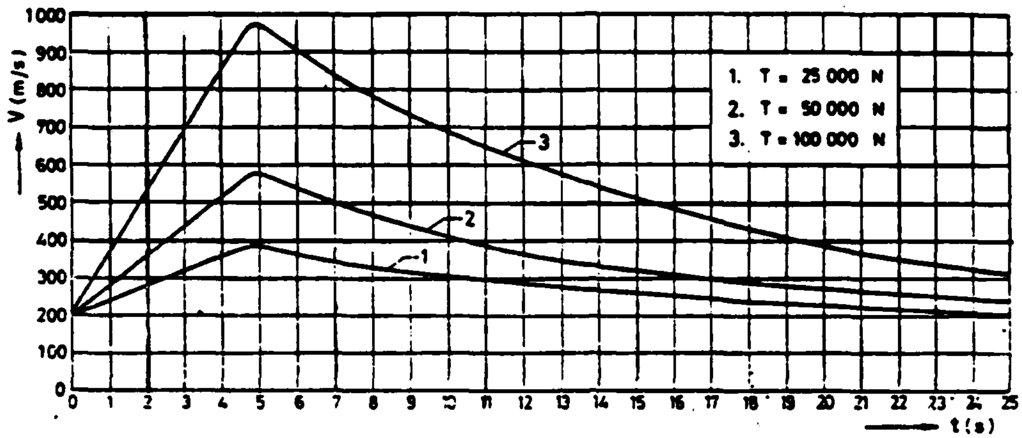


Fig. 7

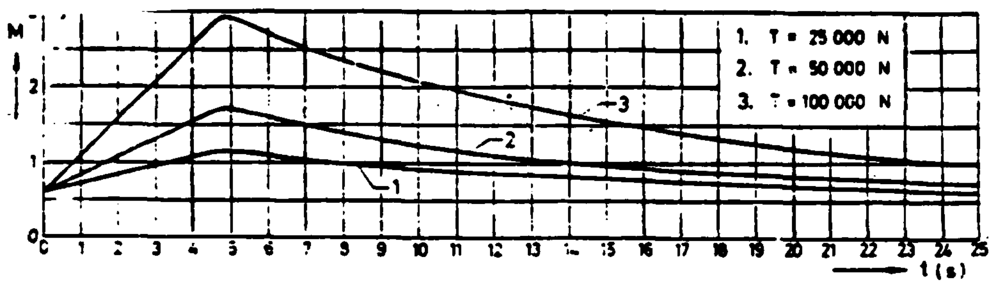


Fig. 8

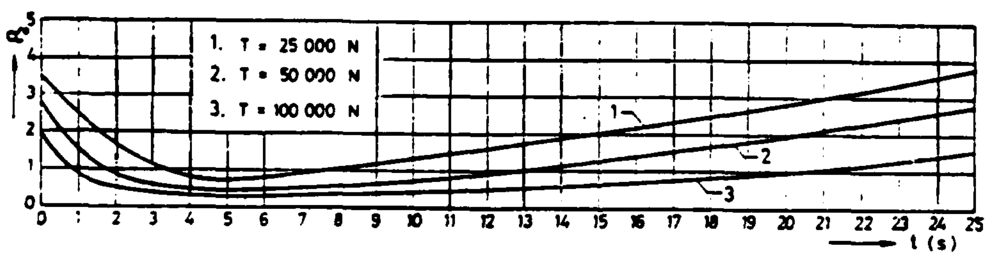


Fig. 9

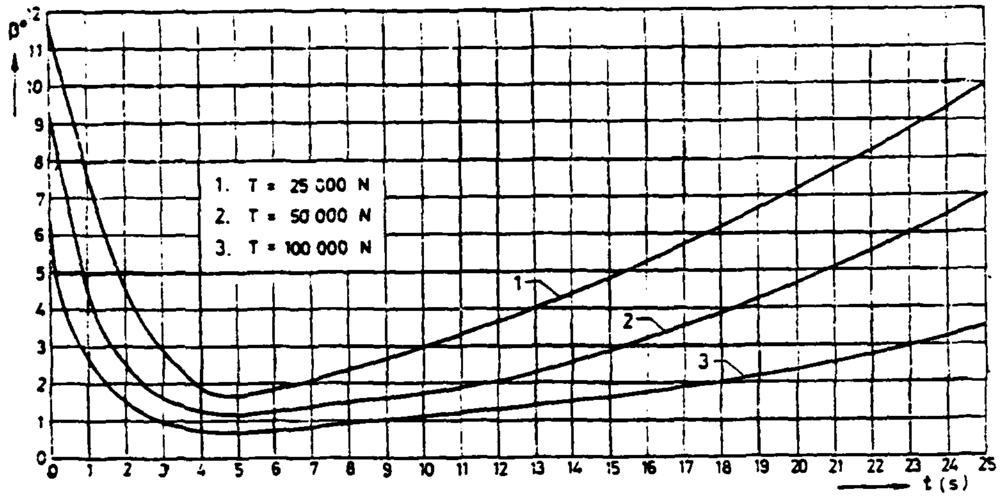


Fig. 10

The variation in time of the attack angle β of the command surfaces, shown in fig. 10, is the command law for the rocket to follow its linear trajectory in the numerical example presented here.

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